TRINITY COLLEGE

Semester One Examination, 2023

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

If required by your examination administrator, please place your student identification label in this box

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	35
Section Two: Calculator-assumed	12	12	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (94 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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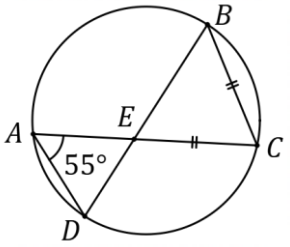
Question 8 **(8 marks)**

(a) The diagram shows the chords AC and BD of a circle intersecting at E. $\angle DAE = 55°$ and $CB = CE$.

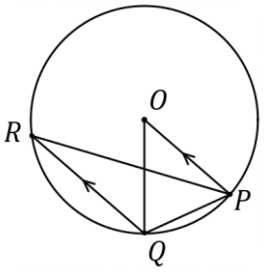
Determine the size of $\angle EBC$ and the size of $\angle ADE$.



(2 marks)

(b) The diagram shows a circle, centre O, and points P, Q and R that lie on it. RQ is parallel to OP.

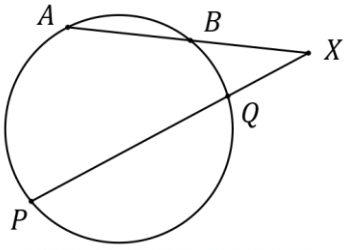
Determine the size of $\angle OQP$ and the size of $\angle RQP$ when $\angle OPR = 18°$.



(3 marks)

(c) In the diagram, secants AB and PQ of a circle intersect at X.

Determine the length AB when $PQ = 8$ cm, $XQ = 4$ cm and $XB = 4.8$ cm.



(3 marks)

Question 9 **(7 marks)**

Let $\underset{\sim}{a} = \binom{k}{2}$ and $\underset{\sim}{b} = \binom{3}{k+1}$. Determine

(a) $\underset{\sim}{b} - \underset{\sim}{a}$ when $k = 1$. (1 mark)

(b) the angle in degrees between the directions of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ when $k = -9$. (1 mark)

(c) the value(s) of k for which $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular. (2 marks)

(d) the value(s) of k for which $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel. (3 marks)

See next page

Question 10 **(10 marks)**

Three coplanar forces act on a small body. One force of 265 N acts on a bearing of $320°$, another of 155 N acts on a bearing of $032°$ and another of 220 N acts on a bearing of $170°$.

Let the perpendicular unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ act in an easterly and northerly direction respectively, and in this question give coefficients correct to 0.01.

(a) Sketch a diagram of the three forces acting on the small body. (1 mark)

(b) Express the force of 155 N using the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$. (2 marks)

(c) Determine the resultant of the three forces, stating its magnitude and the bearing in which it acts. (4 marks)

See next page

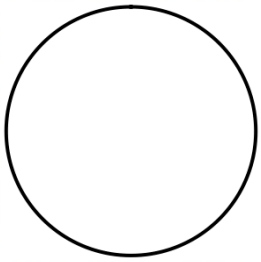
(d) Can the three forces be brought to a state of equilibrium by changing the direction in which the force of 155 N acts? Justify your answer. (3 marks)

See next page

Question 11 **(8 marks)**

A secant from point X meets a circle first at P and then at Q, and a tangent from X touches the same circle at R.

(a) Sketch a diagram to show the secant, tangent and four points. (1 mark)



(b) Prove that $\Delta XQR \sim \Delta XPR$ and hence show that $XP \times XQ = XR^2$. (4 marks)

(c) Determine the exact length of PQ when $XR = 14$ cm and $4 \times PQ = 3 \times XP$. (3 marks)

See next page

Question 12 **(7 marks)**

(a) An unordered selection of 4 dishes must be chosen from 5 hot and 9 cold dishes. Determine the number of ways the selection can be made if

(i) there are no restrictions. (1 mark)

(ii) there must be at least one of each type and at least as many cold dishes as hot. . (2 marks)

(b) Consider the following identity associated with Pascal's triangle:

$$\binom{n}{r+s}\binom{r+s}{r} = \binom{n}{r}\binom{n-r}{s}, \qquad \{n, r, s\} \in \mathbb{Z}^+, n \geq r+s.$$

(i) Show that the identity is true when $n = 8, r = 5$ and $s = 2$. (1 mark)

(ii) Prove the identity is true. (3 marks)

Question 13 **(8 marks)**

In ΔOPQ, R and S are the midpoints of sides OP and OQ respectively. PS intersects QR at T.

(a) Sketch a diagram to show this information. (1 mark)

Let $\underset{\sim}{p} = \overrightarrow{OP}$, $\underset{\sim}{q} = \overrightarrow{OQ}$, $\overrightarrow{PT} = \lambda \times \overrightarrow{PS}$ and $\overrightarrow{QT} = \mu \times \overrightarrow{QR}$.

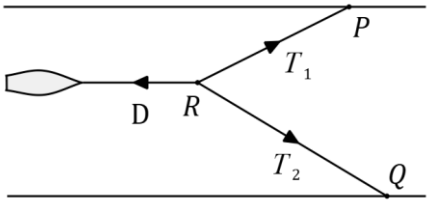
(b) Express $\overrightarrow{OT}$ in terms of $\underset{\sim}{p}, \underset{\sim}{q}$ and λ. (2 marks)

(c) Express $\overrightarrow{OT}$ in terms of $\underset{\sim}{p}, \underset{\sim}{q}$ and μ. (1 mark)

(d) Prove that $\overrightarrow{OP} + \overrightarrow{OQ} = 3 \times \overrightarrow{OT}$. (4 marks)

Question 14 **(8 marks)**

A steady current of 0.6 m/s flows westwards between the parallel banks of a river that are 165 m apart. The diagram below shows a small boat is moored by ropes PR and QR that make angles of $28°$ and $36°$ respectively with the banks of the river.



(a) Determine T_1 and T_2, the tensions in each rope, when the drag force D caused by the water passing the boat is 280 N. (4 marks)

(b) Another small boat leaves Q at a speed of 5 m/s and heads slightly upstream, steering a course that makes an angle of $80°$ with the bank of the river. Determine how far the boat is from P when it reaches the opposite bank, given that P is 20 m downstream from Q. (4 marks)

See next page

Question 15 **(8 marks)**

A charity is selling raffle tickets. The tickets are left over from a previous event and are numbered consecutively from 168 to 500 inclusive.

(a) Determine how many of the tickets have a number that is

(i) a multiple of 8. (2 marks)

(ii) a multiple of 8 or a multiple of 14. (3 marks)

(iii) a multiple of 8 or a multiple of 14 but not a multiple of both. (1 mark)

Once all the tickets have been sold, they will be placed in a large barrel, thoroughly mixed and drawn from the barrel at random and without replacement.

(b) After how many draws can you be certain that at least three of the tickets taken from the barrel have numbers that start with the same digit? Justify your answer. (2 marks)

See next page

Question 16 **(7 marks)**

Six cubes, identical apart from their colour, are to be placed one on top of another to form a tower. Determine the number of different towers that can be made using:

(a) seven different coloured cubes. (1 mark)

(b) a red, a yellow and four blue cubes. (1 mark)

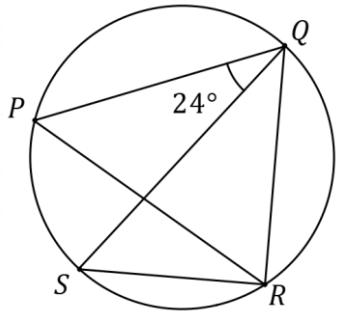
(c) a green, two red and three pink cubes. (1 mark)

(d) one yellow, one red, one blue, one purple, two black and two green cubes. (4 marks)

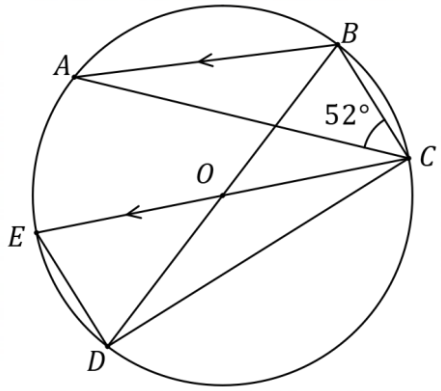
See next page

Question 17 **(6 marks)**

(a) In the diagram, points P, Q, R and S lie on a circle so that QS is a diameter and $PQ = PR$. Determine the size of $\angle RQS$ when $\angle PQS = 24°$. (3 marks)



(b) In the diagram, points A, B, C, D and E lie on a circle with centre O. BD and CE are diameters, and AB is parallel to EC. Determine the size of $\angle OED$ when $\angle BCA = 52°$. (3 marks)



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Question 18 **(9 marks)**

Water in a large river estuary flows at a constant 2.8 km/h on a bearing $070°$. At $1{:}45$ pm, a fishing boat in the estuary has just dropped a crab pot at A and heads off towards B to drop its last one, 1.47 km away on a bearing of $120°$. The fishing boat travels at a constant 8.3 km/h.

(a) Sketch a diagram to represent the sum of the water and boat velocities as it moves directly from A to B, including the angle between their resultant and the water velocity. (2 marks)

(b) Determine the bearing the boat should steer from A to B. (2 marks)

Once the boat reaches B it will spend 5 minutes dropping the last pot and then head off to an anchorage 2.2 km away from B on a bearing of $250°$.

(c) Determine the time at which the boat is expected to reach the anchorage. (5 marks)

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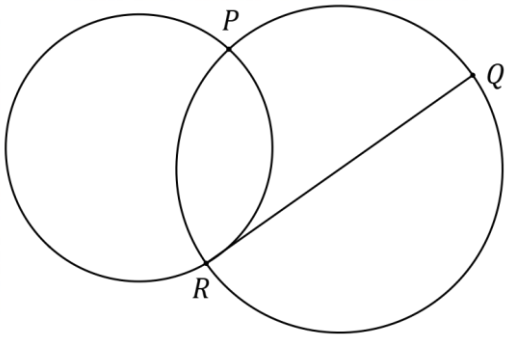
Question 19 **(8 marks)**

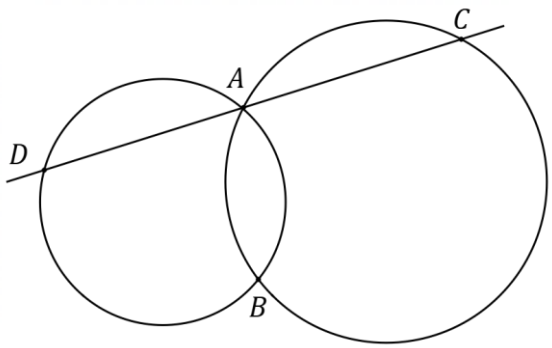
(a) The diagram shows two circles intersecting at P and R. RQ is the diameter of the larger circle.



(i) Add to the diagram a straight line through R, parallel to PQ, that meets the smaller circle at S. (1 mark)

(ii) Prove that PS is a diameter of the smaller circle. (3 marks)

(b) The diagram shows two circles intersecting at A and B. A straight line through A meets one circle at C and the other circle at D.



(i) Add to the diagram a tangent to circle ABC at B that meets circle ABD at E. (1 mark)

(ii) Prove that DE is parallel to BC. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____